Stochastic Uncertainty Quantification of Eddy Currents in the Human Body by Polynomial Chaos Decomposition

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***Abstract*— The finite element method can be used to compute the electromagnetic fields induced into the human body by environmental extremely low frequency fields. However, the electric properties of tissues are not precisely known and may vary depending on the individual, his/her age and other physiological parameters. We propose to account for some uncertainties on the conductivities of the brain tissues and to spread them out to the induced fields by means of a non-intrusive approach based on the chaos Hermite polynomial with the finite element method as a black box [3], [4].**

I. Introduction

The finite element method (FEM) can be used for computing the electric field induced into the human body by extremely low frequencies (ELF) fields [2]. To this aim, the electrical properties of the different tissues (conductivity, permittivity) are required. Unfortunately, these properties are subjected to uncertainty, and their determination is still a matter of discussion. Moreover, most of available data has been obtained from measurements on different animals and extrapolated to humans. Therefore, it is interesting to model and quantify the effect of these uncertainties on the electromagnetic fields induced into the human body. A good statistical characterisation could be obtained by applying a Monte-Carlo (MC) method. It is very time-consuming though (some days in our model). In this work, we use *non-intrusive probabilistic algorithms*, namely a chaos polynomial (CP) approach. Assuming that only the variances of conductivities are finite [3], [4], this kind of method allows to obtain a complete characterization of the induced field in a probabilistic dimension with a reduced computational cost, a few hours in comparison to MC simulations.

II. Incorporation of Stochastic Uncertainty

A. Deterministic framework

In the deterministic context of ELF magnetic fields, it is generally assumed that the reaction field due to eddy currents in living tissues is negligible. This allows to develop specific formulations, where the computational domain Ω is limited to the human body. In this work we use the $\phi - \mathbf{a}$ FE formulation with the electric scalar potential ϕ to determine and the magnetic vector potential $\mathbf{a}$ (magnetic flux density $\mathbf{b} = \operatorname{curl}\mathbf{a}$) known *a priori*, as the reaction field is negligible.

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By weakly imposing $\operatorname{div}\mathbf{j} = 0$ with the boundary condition $\mathbf{n}\cdot\mathbf{j}|_{\partial\Omega} = 0$, the $\phi - \mathbf{a}$ formulation reads [2]:

$$(\sigma\, \partial_t \boldsymbol{a}, \operatorname{grad}\phi')_\Omega + (\operatorname{grad}\phi, \operatorname{grad}\phi')_\Omega = 0\,, \quad \forall \phi' \in \mathrm{H}(\operatorname{grad},\Omega)\,. \tag{1}$$

As human model, we consider the the phantom ZOL [2] available from the Visible Human project [1]. The electrical conductivity values σ are taken from [5]. The flux density source is a 50 Hz uniform vertical field $b_z = 1\,\mathrm{mT}$. In the literature concerning the protection against ELF fields, three scalar values are generally computed from the current density $\mathbf{j}$ for each organ: the (spatial) average j_A, the maximum value j_M, and the 99% percentile [6]. In particular, the ICNIRP recommendations focus on j_M. Similar definitions exist for the electric field $\mathbf{e}$.

B. Uncertainties

In this paper, we are mainly concerned with the fields induced in the head. The conductivities in the brain $\sigma_B(\omega)$, and of the cerebellum $\sigma_C(\omega)$ are modeled within a probabilistic framework, by assuming that they are random variables. Therefore, the average j_A, the maximum j_M or the 99% percentiles j_P of the induced fields are random as well. In particular, by applying the maximum entropy principle [7] we model (arbitrarily) $\sigma_B(\omega)$ and $\sigma_C(\omega)$ (in S/m) as independent random variables, uniformly distributed between the $1/3$ and three times the values in [5], i.e.

$$\sigma_B(\omega) \sim U([0.0178\ ;\ 0.160])\,, \tag{2}$$

$$\sigma_c(\omega) \sim U([0.0317\ ;\ 0.286])\,. \tag{3}$$

C. The Non -Intrusive Approach

As the conductivities of the brain and the cerebellum are two independent random variables of finite variance, we can expand them as a truncated series of order p_{in} in the two-dimensional Hermite polynomials of a random gaussian vector $\xi(\omega) = (\xi_1(\omega), \xi_2(\omega))$, known as *Hermite chaos polynomials* [4]:

$$\sigma_B(\omega) \approx \sum_{i=0}^{P_{in}} \sigma_{Bi}\psi_i(\xi(\omega))\,, \tag{4}$$

$$\sigma_C(\omega) \approx \sum_{i=0}^{P_{in}} \sigma_{Ci}\psi_i(\xi(\omega))\,, \tag{5}$$

where σ_{Bi} and σ_{Ci} are scalar values that depend on the probabilistic law of the conductivities, $P_{in} = C_{2+p_{in}}^{p_{in}}$ is the number of two-dimensional polynomial of order smaller than p_{in}, and ψ_i is the i^{th} two-dimensional Hermite polynomial.

The stochastic problem is solved by means of a CP decomposition of both the conductivity and the induced fields [4]. There is only an assumption on the conductivities, with no *a priori* hypothesis on the probabilistic distribution shape of the average j_A, the maximum j_M or the 99% percentiles j_P of the induced fields.

Those induced fields – the average current density in the brain $\mathbf{j_A}(\omega) = \mathbf{j_A}(\xi(\omega))$ – are calculated by the FEM from any pair of values $(\sigma_B(\xi(\omega)), \sigma_C(\xi(\omega)))$. The average density belongs to a space that can be spanned by the polynomials $\psi(\xi(\omega))$ and can thus be written as a truncated series to an order p_{out}:

$$\mathbf{j_A}(\omega) = \sum_{i=0}^{P_{out}} \mathbf{j_A}^m \psi_m(\xi(\omega))\,. \tag{6}$$

The unknown real coefficients $\mathbf{j_A}^m$ are computed taking into account the orthogonality properties of the Hermite polynomials:

$$\mathbf{j_A}_i^m = \frac{\mathrm{E}[\mathbf{j_A}(\omega)\psi_m(\xi(\omega))]}{\mathrm{E}[\psi_m(\xi(\omega))^2]}, \tag{7}$$

with $\mathrm{E}[\cdot]$ the mathematical expectation. The denominator can be computed analytically. A Hermite Gauss integration scheme with d points is used to determine the numerator, which is an integral [4]:

$$\mathrm{E}[\mathbf{j_A}(\omega)\psi_m(\xi(\omega))] \approx \sum_{i=1}^{d} \cdots \sum_{j=1}^{d} w_{i,j}(\mathbf{j_A}((t_1,t_2)_{i,j}))\psi_m((t_1,t_2)_{i,j}), \tag{8}$$

where $(t_1, t_2)_{i,j}$ is the i, j-th Gauss point, and $w_{i,j}$ the associated weight in the two-dimensional Cartesian rule. Therefore, the deterministic problem must be solved d^2 times, with the conductivity evaluated through (4) and $(\xi_1(\omega), \xi_2(\omega)) = (t_1, = t_2)_{i,j}$, $i, j = 1, \ldots, d$.

III. Results and Discussion

The non-intrusive polynomial chaos decomposition methods are governed by three parameters: p_{in} determines the accuracy of the approximation made on the input random variables; p_{out} is the order of truncation of the induced fields (maximum, average and percentiles of the electric field or current density) and d is the number of integration points. We adopt $p_{in} = 16$, $p_{out} = 6$ or 10, $d = 14$.

The probabilistic density of the average j_A, the 99% percentile j_P and the maximum j_M current density in the brain are depicted in Fig. 1 for $p_{out} = 6$ and 10. The convergence of the non-intrusive method can be clearly observed. The mean and standard deviation of j_A are $1.2\,10^{-4}$ A/m^2 and $4.3\,10^{-5}$ A/m^2, respectively. The probabilistic density of j_A presents the sharpest peak among the three considered quantities. Note that the values higher than the mean are more probable than the rest. With regard to the 99% percentile j_P in the white and grey matter of the brain, the mean is $1.9\,10^{-4}$ A/m^2 with a standard deviation of $6.8\,10^{-5}$ A/m^2. It is more dispersed around the mean than the j_A. Once again the right tail is slightly more probable than the left one. Finally, the maximum j_M in the brain presents a mean of $3.2\,10^{-4}$ A/m^2 and a standard deviation of $1.3\,10^{-4}$ A/m^2. This last parameter is the most spread (the standard deviation and the mean have the same order of magnitude and the probabilistic density is the largest). It is clear that the probability to be above $3\,10^{-4}$ A/m^2 is nearly 0 for j_A and j_P. It is not negligible anymore for j_M. Note that the maximum value of computed fields may be (strongly) influenced by the FE mesh quality.

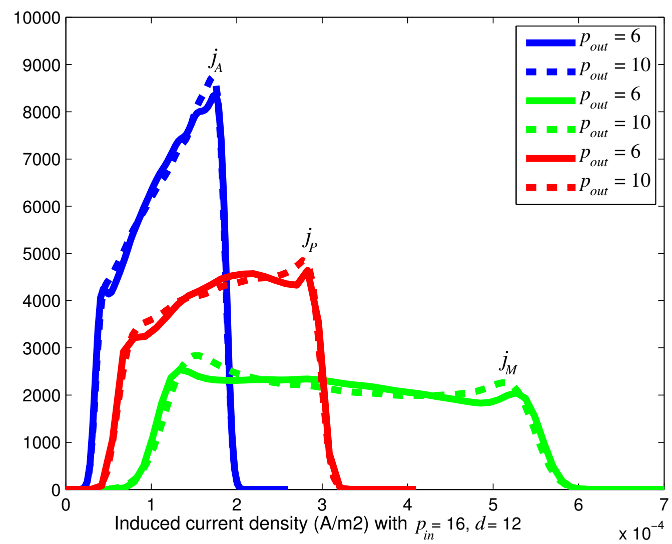

Fig. 1. Probabilistic density of the averaged (j_A), 99-percentile (j_P) and maximum (j_M) current density in the brain

Further details on the non-intrusive probabilistic method will be provided in the full paper. Special attention will be paid to the influence of p_{in}, d and p_{out} and some probability threshold. Results on the induced electric field and in the cerebellum will be presented as well.

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